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THE CURTIS—GODSON APPROXIMATION FOR CALCULATING THE RADIATION OF A NONISOTHERMAL GAS

Yu. A. Popov

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We obtain a criterion for the applicability of the Curtis—Godson approximation. Using this approximation, we make calculations of the total heat fluxes for nonisothermal steam and carbon dioxide.

Until now no conditions have been obtained for the applicability of the frequently used Curtis—Godson approximation [2, 3]. The more homogeneous and isothermal a medium is, the more accurate the approximation will be. In [2] it was shown, using the example of the transfer problem in a narrow spectrum interval, that the error of the Curtis—Godson approximation may be considerable.

The law of transmission of radiation averaged over a narrow spectrum interval, when we use a statistical model of the absorption bands of a gas, has the form [3]

$$\langle D_\nu \rangle = e^{-\xi_\nu}, \tag{1}$$

where

$$\xi_\nu = \frac{1}{\delta} \int_{-\infty}^{\infty} \frac{a_{\nu\nu'}}{1 + a_{\nu\nu'}} d\nu', \tag{2}$$

$$a_{\nu\nu'} = \frac{\delta}{\pi} \int_0^l \tilde{K}_\nu(z) \frac{\alpha(z)}{(\nu - \nu')^2 + \alpha^2(z)} dz, \tag{3}$$

δ is the average distance between the lines in the band; α is the width of a line (the Lorentz form of the lines); $\tilde{K}_\nu$ is the absorption coefficient of the gas averaged over a narrow spectrum interval. We set

$$\alpha(z) = \bar{\alpha} + \alpha'(z). \tag{4}$$

Substituting (4) into (3), we obtain

$$a_{\nu\nu'} = \frac{\delta}{\pi} \frac{\bar{\alpha}}{(\nu - \nu')^2 + \bar{\alpha}^2} \left[\int_0^l \tilde{K}_\nu(z)\, dz + \int_0^l \frac{\alpha'(z)}{\bar{\alpha}} \tilde{K}_\nu(z)\, dz - \frac{2\bar{\alpha}^2}{(\nu - \nu')^2 + \bar{\alpha}^2} \int_0^l \frac{\alpha'(z)}{\bar{\alpha}} \tilde{K}_\nu(z)\, dz + O\left(\int_0^l \left(\frac{\alpha'}{\bar{\alpha}} \right)^2 \tilde{K}_\nu(z)\, dz \right) \right]. \tag{5}$$

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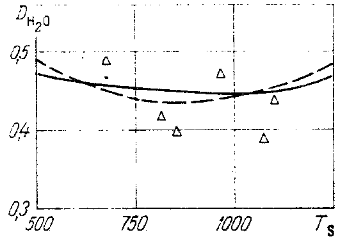

Fig. 1

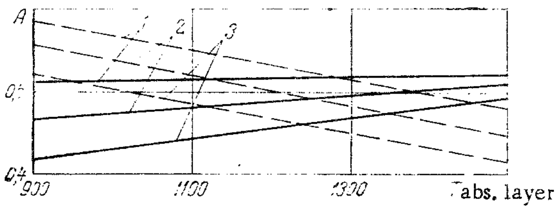

Fig. 2

Fig. 1. Transmissivity of a layer of steam as a function of its temperature. The solid curve represents calculation by the method of [1], the dashed curve by the Curtis–Godson approximation, and the points marked by triangles are taken from the results of the experiments in [5]. T_s, °K.

Fig. 2. Absorptivity of a layer of gas as a function of its temperature: 1) 1100°C; 2) 1300; 3) 1500. The dashed curves represent the experiment of [6]; the solid curves represent the calculation in the Curtis–Godson approximation. $T_{abs.layer}$, °C.

We select the quantity $\alpha'(z)$ from the condition

$$\int_0^l \alpha'(z)\tilde{K}_\nu(z)\,dz = 0. \tag{6}$$

From (1), (6), we have

$$\bar{\alpha} = \int_0^l \alpha(z)\,\tilde{K}(z)\,dz \Big/ \int_0^l \tilde{K}(z)\,dz. \tag{7}$$

The expression (7) gives us the best average for the width of a line. From (5), (6) it follows that

$$a_{\nu\nu'} = \frac{\delta}{\pi}\,\frac{\bar{\alpha}}{(\nu-\nu')^2+\bar{\alpha}^2}\int_0^l \tilde{K}_\nu(z)\,dz. \tag{8}$$

The expressions (7), (8) form the basis of the Curtis–Godson approximation. In this approximation the problem reduces to an isothermal problem if we make the substitutions

$$\tilde{K}l \to \int_0^l \tilde{K}dz \text{ и } \alpha \to \bar{\alpha}.$$

From (5) we obtain the condition for the applicability of this approximation:

$$\int_0^l \left(\frac{\alpha'(z)}{\bar{\alpha}}\right)^2 \tilde{K}_\nu(z)\,dz \Big/ \int_0^l \tilde{K}_\nu(z)\,dz \ll 1,$$

which can be written in the form

$$\left\langle\left(\frac{\alpha(z)-\bar{\alpha}}{\bar{\alpha}}\right)^2\right\rangle \ll 1. \tag{9}$$

On the basis of the results of [4], we prepared a program for calculating the total flux of radiation from a nonisothermal mixture of carbon dioxide and steam in the Curtis–Godson approximation. The radiation was produced by a layer of steam at 1000°K. The product of the H_2O partial pressure and the thickness of the layers is the same in each case, 10 cm·atm. In Fig. 1 the results of our calculation are compared with the results of the more exact calculations of [1] and with the data obtained experimentally in [5]. The data are in good agreement. In Fig. 2 the results of our calculations in the Curtis–Godson approximation are compared with the data of [6] for a mixture of carbon dioxide and steam (two-layer problem). The two layers of gas have the same composition: H_2O partial pressure 0.184 atm, CO_2 partial pressure 0.096 atm. The thickness of the layers is 10 cm. The maximum discrepancy between the calculated and experimental data is 20%.

NOTATION

D, transmissivity; A, absorptivity; ν, wave number; z, coordinate; l, path length of beam in gas; α, width of line; $\bar{\alpha}$, average value of width of line; δ, average distance between lines in gas band; $\tilde{K}_{\nu}$, absorption coefficient of gas averaged over narrow spectrum interval.

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